On the Grayson-Stuhler Filtration of Euclidean Lattices

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In this instructional talk I will introduce the notion of semistability for Euclidean lattices, and define the canonical filtration of a Euclidean lattice by semi stable ones. This filtration has many remarkable properties, and has probably not received all the attention it deserves. This theory dates back to Stuhler ([Stu76]) and Grayson ([Gra84]), who used it to build an alternative to Borel Serre's compactification of locally symmetric spaces.

In short, given a Euclidean lattice L, one can plot in the plane the points $(\dim M, \log \det M)$ as M varies among all sublattices of L. Their convex hull is bounded below by a certain convex polygon which has two important properties :

- each of its vertices corresponds to a unique sublattice M of L.
- these sublattices form a chain, which we call the Grayson-Stuhler filtration of L.

Among other noteworthy properties, this filtration is invariant under automorphisms and scalar extension. As time allows, I will also speak about a conjecture regarding the behaviour of this filtration with respect to tensor product.

References

[Cas04] Bill Casselman, *Stability of lattices and the partition of arithmetic quotients*, Asian J. Math. **8** (2004), no. 4, 607–637.

[Gra84] Daniel R. Grayson, *Reduction theory using semistability*, Comment. Math. Helv. **59** (1984), no. 4, 600–634. MR 780079 (86h:22018)

[Stu76] Ulrich Stuhler, *Eine Bemerkung zur Reduktionstheorie quadratischer Formen*, Arch. Math. (Basel) **27** (1976), no. 6, 604–610.